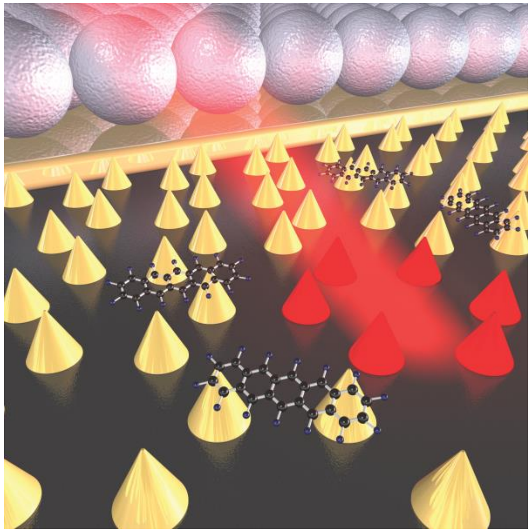
Basic module
Physics of Nanostructures

Summer term 2026

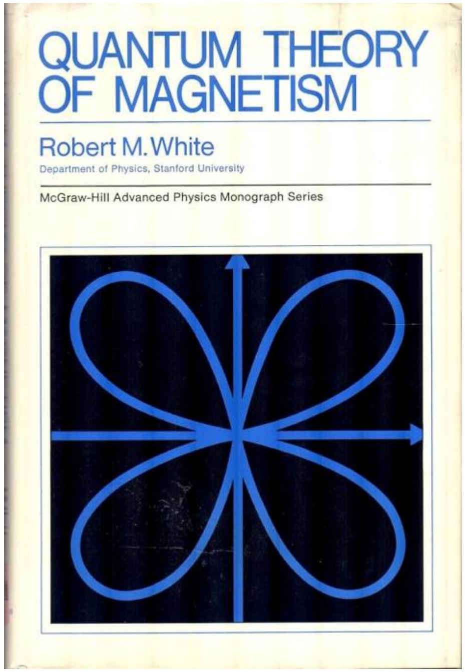


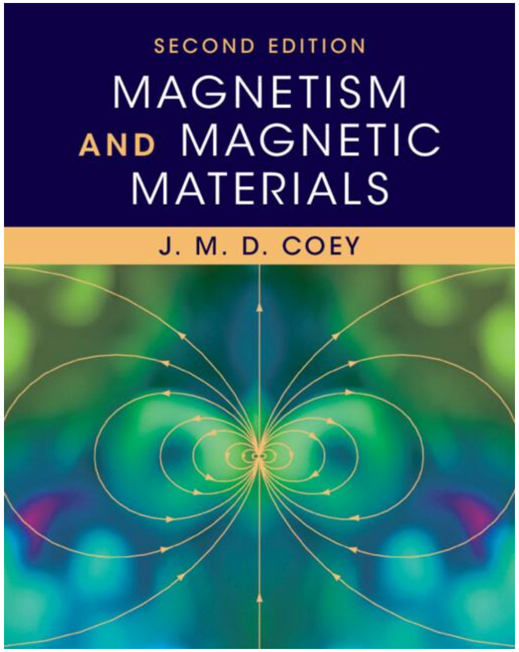
M. Fleischer, I. Zaluzhnyy,
Excercises: R. Löffler

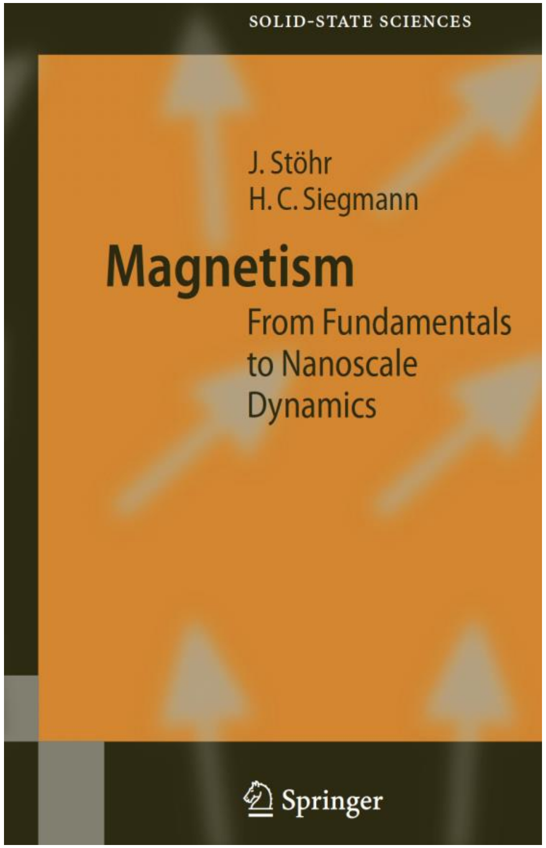
Literature

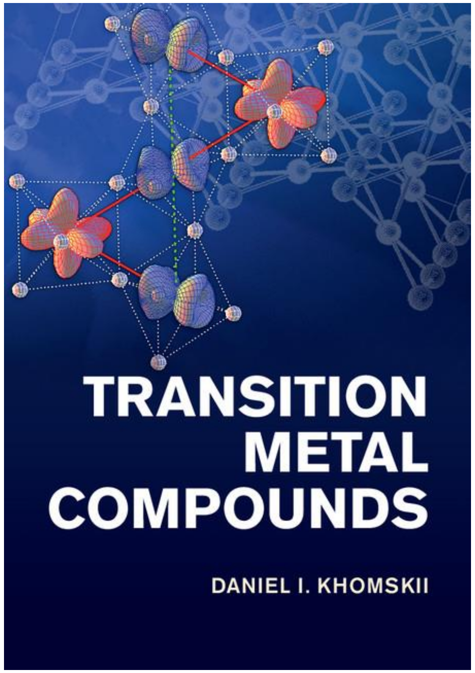
J.M.D. Coey, *Magnetism and magnetic materials*, 2026
J. Ströhr, H.C. Siegmann, *Magnetism*, 2006

R.M. White, *Quantum theory of magnetism,* 1983
D.I. Khomskii, *Transtion metal compounds,* 2014







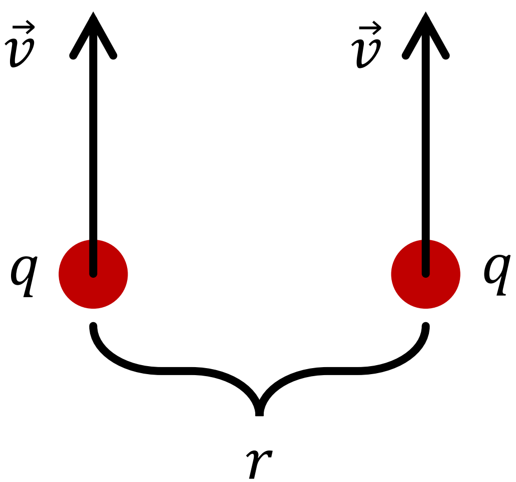


How strong are magnetic forces

the Coulomb law $\vec{E} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \cdot \vec{r}}{r^3}$

the Biot-Savart law $\vec{B} = \frac{\mu_0}{4\pi} \cdot \frac{\vec{r} \times \vec{j}}{r^3}$

the Lorentz force $\vec{F} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$



$$F_e \sim \frac{1}{4\pi\varepsilon_0} \cdot \frac{q^2}{r^2}$$

$$F_m \sim \frac{\mu_0}{4\pi} \cdot \frac{q^2 \cdot v^2}{r^2}$$

$$\frac{F_m}{F_e} \sim \mu_0 \varepsilon_0 v^2 \sim \left(\frac{v}{c}\right)^2$$

Maxwell's equations

in vacuum

$$\left\{\begin{array}{ll} \nabla \cdot \vec{B} = 0 & \nabla \cdot \vec{E} = \dfrac{\rho}{\varepsilon_0} \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} & \nabla \times \vec{B} = \mu_0 \vec{J} + \dfrac{1}{c^2}\dfrac{\partial \vec{B}}{\partial t} \end{array}\right.$$

Electric displacement field $\vec{D} = \varepsilon_0 \vec{E} + \vec{P} = \varepsilon_0 (1 + \chi)\vec{E} = \varepsilon \cdot \vec{E}$

Magnetic field intensity $\vec{H} = \dfrac{1}{\mu_0}\vec{B} - \vec{M}$

$$\vec{B} = \mu_0\left(\vec{H} + \vec{M}\right) = \mu_0 (1 + \chi)\vec{H} = \mu \cdot \vec{H}$$

in matter

$$\left\{\begin{array}{ll} \nabla \cdot \vec{B} = 0 & \nabla \cdot \vec{D} = \rho_f \\ \nabla \times \vec{E} = -\dfrac{\partial \vec{B}}{\partial t} & \nabla \times \vec{H} = \vec{J}_f + \dfrac{\partial \vec{D}}{\partial t} \end{array}\right.$$

The origin of magnetic properties

$\vec{B} = 0$

$$\hat{H}_0 = \sum \frac{\hat{\vec{p}}^2}{2m_e} + U(\vec{r})$$

$\vec{B} = \text{const} \neq 0$

$\vec{B} = \nabla \times \vec{A}$

$\vec{A} = \frac{1}{2}\vec{B} \times \vec{r}$

$$\mu_B = \frac{e}{m_e} \cdot \frac{1}{2}\hbar = 9.27 \cdot 10^{-24} \left[\frac{\text{J}}{\text{T}}\right]$$ the Bohr magneton

$$\hat{H} = \hat{H}_0 + \hat{V} = \sum \frac{1}{2m_e}\left(\hat{\vec{p}} - e\vec{A}\right)^2 + U(\vec{r}) - 2\mu_B \cdot \hat{\vec{S}}\vec{B}$$

$$\hat{V} = \underbrace{\sum \frac{e^2\left[\vec{B} \times \vec{r}\right]^2}{8m_e}}_{\text{orbital diamagnetism}} \underbrace{- \mu_B\left(\hat{\vec{L}} + 2\hat{\vec{S}}\right)\vec{B}}_{\text{Zeeman energy}}$$

The origin of magnetic properties

Atoms with $\hat{\vec{L}} = 0$, $\hat{\vec{S}} = 0$: only orbital diamagnetism

$$\Delta E = \sum \frac{e^2}{8m_e} \left\langle \left[\vec{B} \times \vec{r}\right]^2 \right\rangle$$

$$\Delta E = -\vec{m}\vec{B} \approx \frac{e^2 B^2}{12 m_e} Z\langle r^2 \rangle$$

$$\vec{m} = -\frac{\partial \Delta E}{\partial B} = \chi \vec{H}$$

$$\chi = -\frac{\mu_0 e^2}{6 m_e} Z\langle r^2 \rangle$$

Atoms with $\hat{\vec{J}} = \hat{\vec{S}} + \hat{\vec{L}} \neq 0$: Langevin paramagnetism

$$m = g\mu_B J$$

$$g = \begin{cases} 1, if \ \hat{\vec{L}} \neq 0 \ and \ \hat{\vec{S}} = 0 \\ 2, if \ \hat{\vec{L}} = 0 \ and \ \hat{\vec{S}} \neq 0 \\ 1 + \dfrac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \end{cases}$$

the Landé g-factor

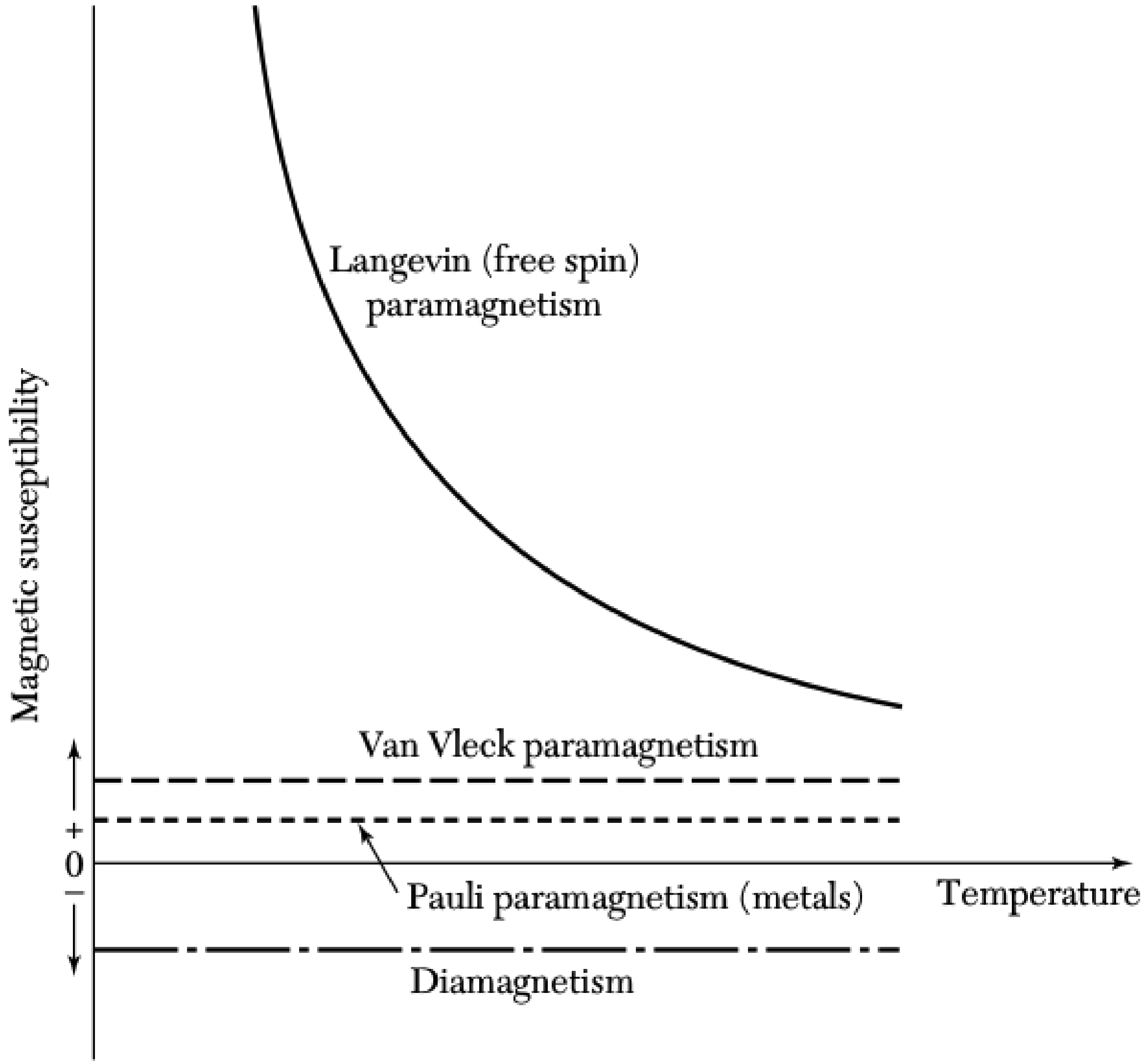
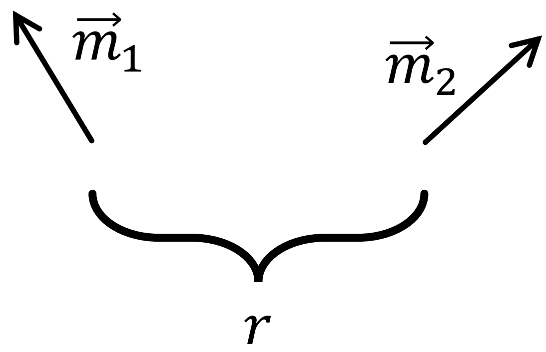


Figure 1 Characteristic magnetic susceptibilities of diamagnetic and paramagnetic substances.

C. Kittel, *Introduction to solid state physics*

Interaction between localized moments

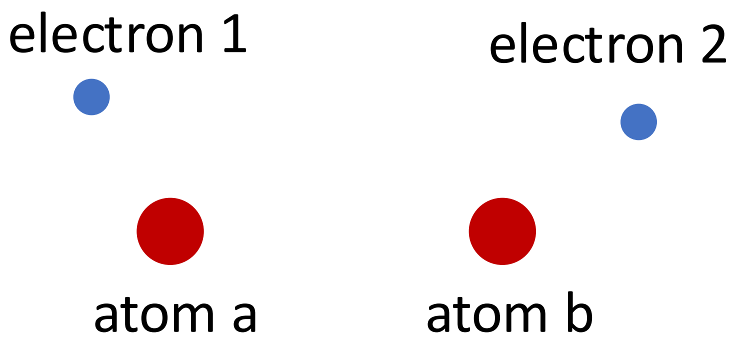


$$\vec{B}_2 = \frac{\mu_0}{4\pi} \cdot \left(\frac{3(\vec{m}_1 \vec{r})\vec{r}}{r^5} - \frac{\vec{m}_1}{r^3} \right)$$

$$U = -\vec{m}_2 \vec{B}_2 = \frac{\mu_0}{4\pi} \cdot \left(\frac{\vec{m}_1 \vec{m}_2}{r^3} - \frac{3(\vec{m}_1 \vec{r})(\vec{m}_2 \vec{r})}{r^5} \right) \sim \frac{\mu_0}{4\pi} \cdot \frac{\mu_B^2}{a_B^3} \sim 5\ K$$

Direct dipole-dipole interaction is too weak to explain the magnetic properties up to the Curie temperature $T_C \sim 10^2 K$

Exchange interaction



the Coulomb law $V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{|\vec{r}_1 - \vec{r}_2|}$

$$\Delta E = \iint \psi^*(\vec{r}_1, \vec{r}_2) \cdot V \cdot \psi(\vec{r}_1, \vec{r}_2) d\vec{r}_1 d\vec{r}_2$$

If spins are anti-parallel ($S = 0$)

$$\psi_0(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}[\psi_a(\vec{r}_1)\psi_b(\vec{r}_2) + \psi_a(\vec{r}_2)\psi_b(\vec{r}_1)]$$

If spins are parallel ($S = 1$)

$$\psi_1(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}[\psi_a(\vec{r}_1)\psi_b(\vec{r}_2) - \psi_a(\vec{r}_2)\psi_b(\vec{r}_1)]$$

$\Delta E = A \pm J$

$$A = \frac{e^2}{4\pi\varepsilon_0} \iint \frac{|\psi_a(\vec{r}_1)|^2 |\psi_b(\vec{r}_2)|^2}{|\vec{r}_1 - \vec{r}_2|} d\vec{r}_1 d\vec{r}_2 \sim \frac{e^2}{4\pi\varepsilon_0} \cdot \frac{1}{3a_B} \sim 10^5\ K$$

Exchange interaction

$\Delta E = A \pm J$ (+ for $S = 0$; - for $S = 1$)

$$J = \frac{e^2}{4\pi\varepsilon_0} \cdot 2\Re e \iint \frac{\psi_a^*(\vec{r}_1)\psi_b^*(\vec{r}_2)\psi_a(\vec{r}_2)\psi_b(\vec{r}_1)}{|\vec{r}_1 - \vec{r}_2|} d\vec{r}_1 d\vec{r}_2$$

$$\psi(r) \sim e^{-\frac{r}{a_B}}$$

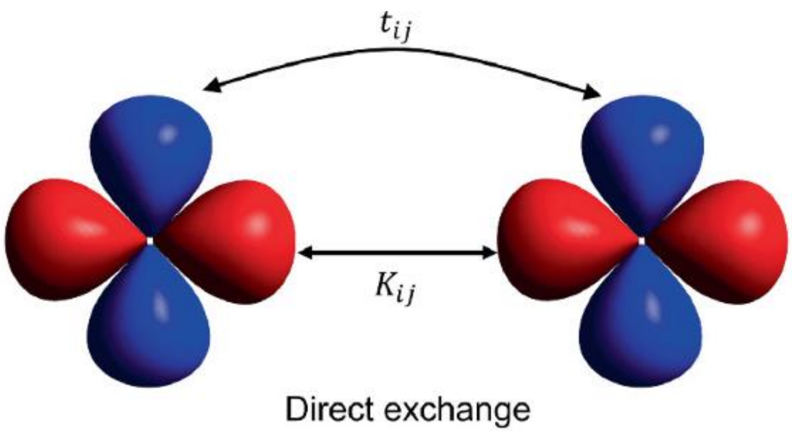
$$J \sim \frac{e^2}{4\pi\varepsilon_0} \cdot 2 \cdot e^{-2\frac{r_{ab}}{a_B}} \frac{1}{a_B} \sim 400\ K$$ exchange energy

if $J > 0 \Longrightarrow$ min. energy for $S = 1 \Longrightarrow$ ferromagnetic order ↑ ↑ ↑ ↑ ↑ ↑

if $J < 0 \Longrightarrow$ min. energy for $S = 0 \Longrightarrow$ antiferromagnetic order ↑ ↓ ↑ ↓ ↑ ↓

Exchange interactions

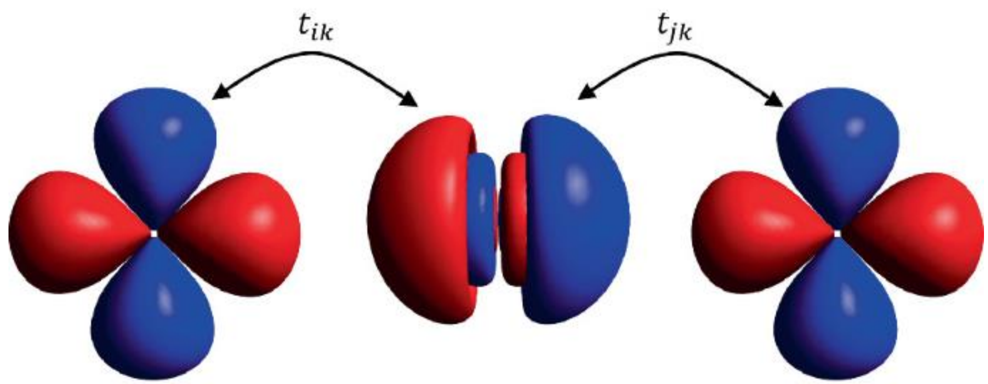
Direct exchange (some pure metals): direct exchange between 3d-electrons



G. Rajaraman, *Computational modelling of molecular nanomagnets*

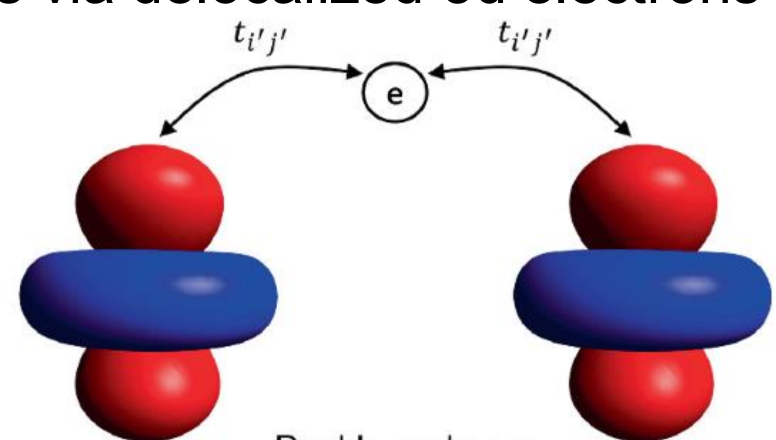
Exchange interactions

Superexchange (e.g. MnO): indirect exchange between 3d-electrons of metal via 2p-electrons of oxygen



Superexchange

Double exchange (e.g. $La_{1-x}Sr_xMnO_3$): indirect exchange between localized 3d-electrons via delocalized 3d electrons



Double exchange

G. Rajaraman, *Computational modelling of molecular nanomagnets*

Exchange interactions

RKKY interaction (e.g. rare-earths): indirect exchange between very localized 4f-electrons via 5d/6s conduction electrons

Ruderman
Kittel
Kasuya
Yosida

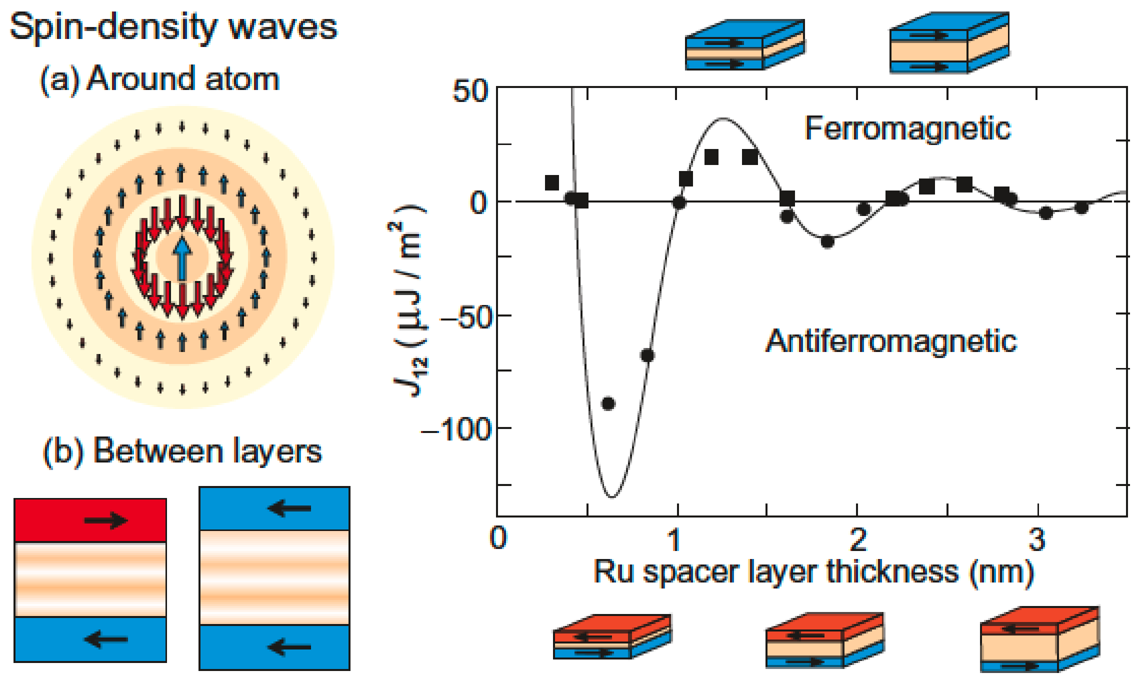


Fig. 7.34. (a) Spin polarization of the conduction electrons around a localized magnetic impurity, showing the characteristic RKKY oscillations given by (7.46). (b) Spin polarization of electrons between two magnetic layers. The relative magnetization alignment in the two layers depends of the distance between the layers, and is caused by induced spin polarization in the "nonmagnetic" spacer layer, the sign of which is distance-dependent. Right: Interlayer exchange coupling strength J_{12} between two ferromagnetic $Ni_{80}Co_{20}$ layers across a Ru spacer layer of variable thickness [312]. The experiment utilized a specially engineered multilayer structure.

J.M.D. Coey, *Magnetism and magnetic materials*, 2026

The Heisenberg model

$$\hat{H} = \underbrace{-\sum_{i\neq j} J(\vec{r}_i - \vec{r}_j)\hat{\vec{S}}_i\hat{\vec{S}}_j}_{\text{Exchange energy}} \underbrace{- g\mu_B\vec{B}\sum_{i}\hat{\vec{S}}_i}_{\text{Zeeman energy}}$$

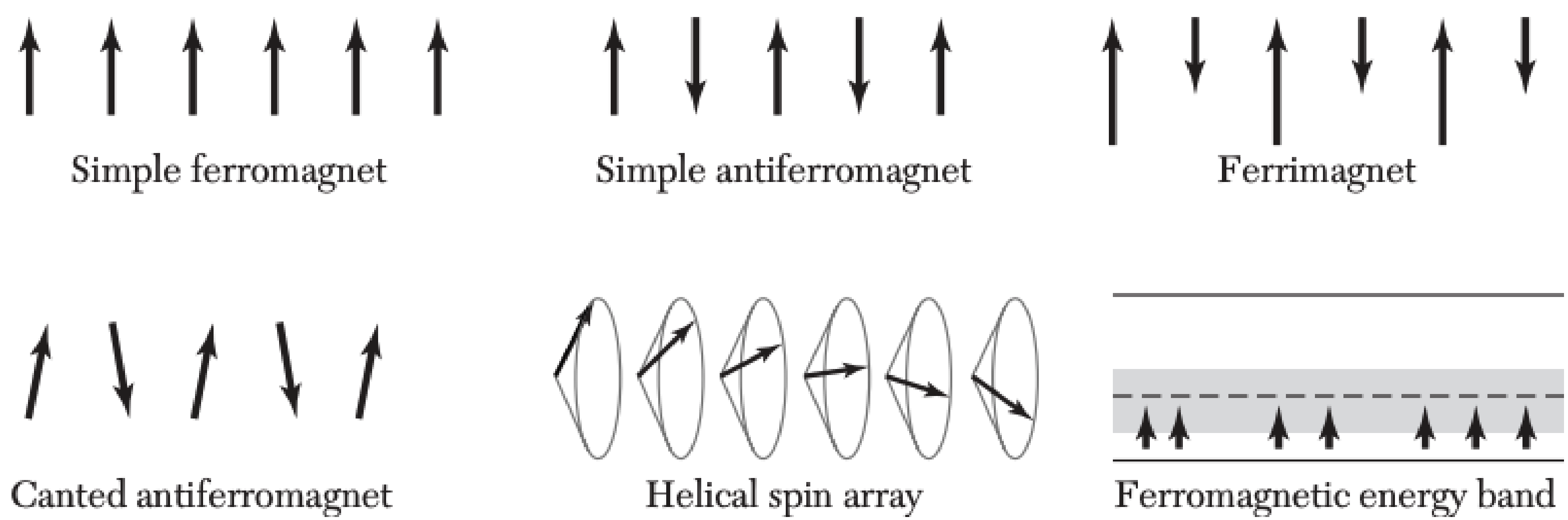
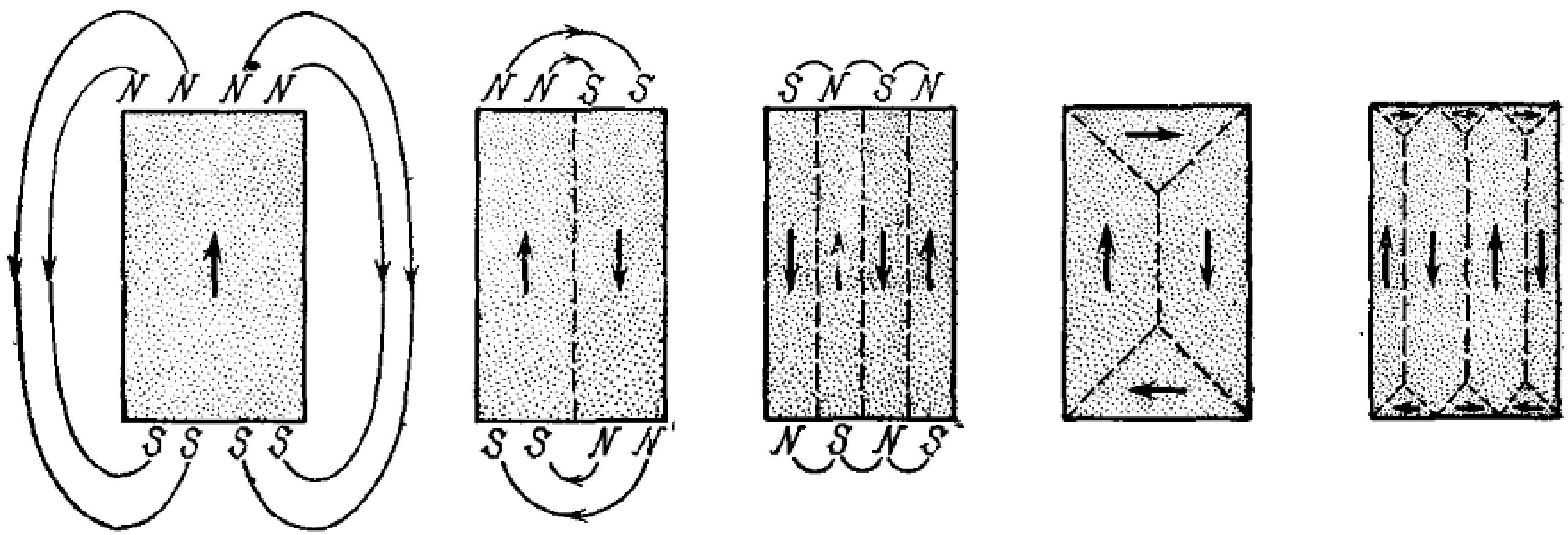


Figure 1 Ordered arrangements of electron spins.

C. Kittel, *Introduction to solid state physics*

Magnetic domains



C. Kittel, *Introduction to solid state physics*

Domain walls

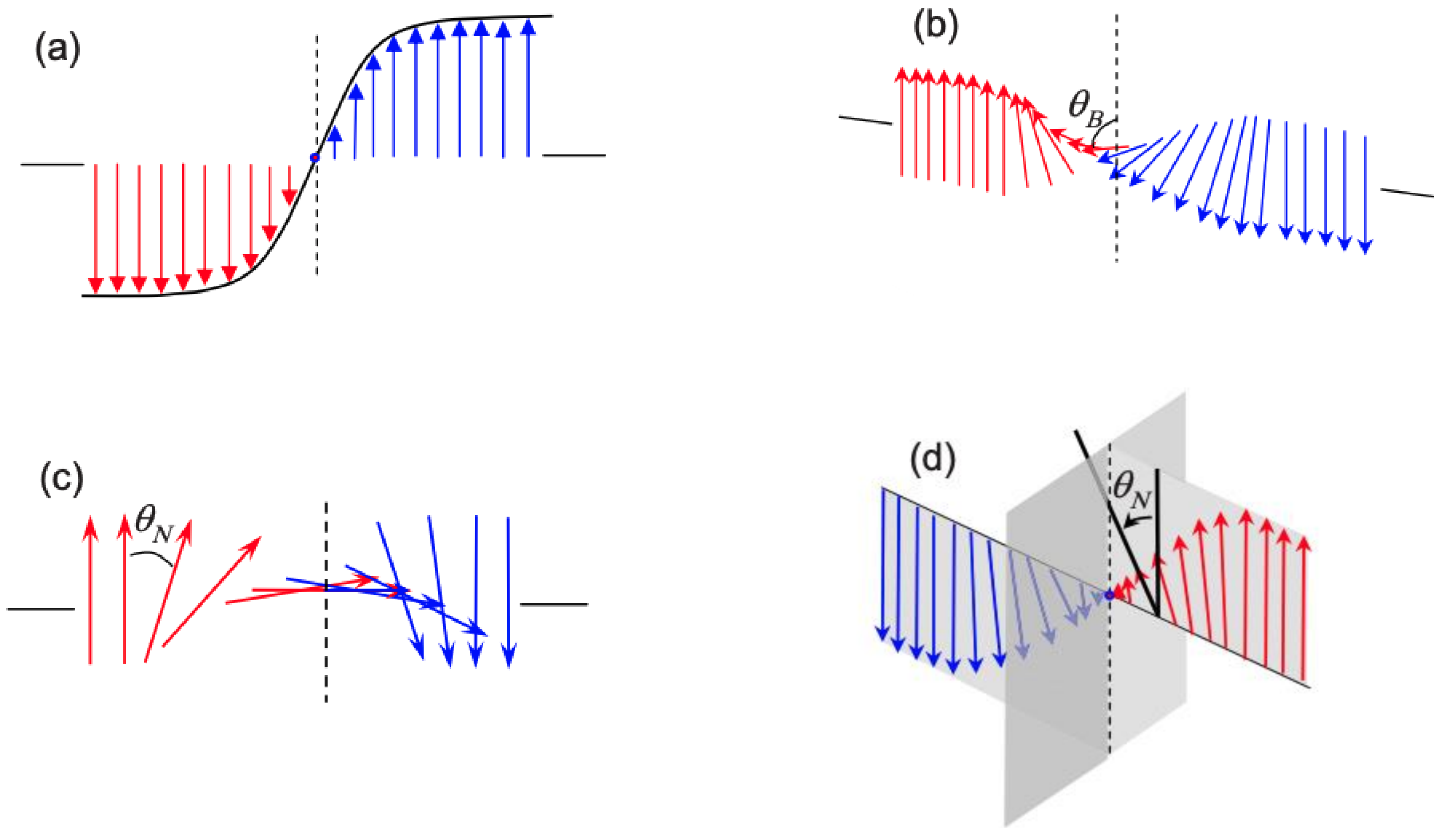


FIG. 1. (Color online) Different types of domain walls: (a) Ising type, (b) Bloch type, (c) Néel type, and (d) Mixed Ising-Néel type walls. A mixed Ising-Bloch type would look similar to (d) except that the rotation (θ_B) would be out of the plane of the polarization vector.

D. Lee et al, *Phys. Rev. B 8**0**, 060102(R) (2009)*