1. Introduction to magnetism

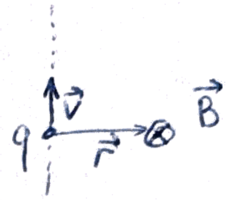
Reminder: - Electric field ($\vec{E}$) is produced by electric charges:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q\cdot\vec{r}}{r^3}\left[\frac{V}{m}\right] \leftarrow \text{the Culomb law}$$

$$\varepsilon_0 \approx 8.85\cdot10^{-12}\left[\frac{F}{m}\right] \leftarrow \text{the vacuum electric permittivity}$$

- Magnetic field ($\vec{B}$) is produced by moving charges (currents):

$$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi}\cdot\frac{\vec{r}\times\vec{j}}{r^3}\,[T] \leftarrow \text{the Biot-Savart law}$$



$$\mu_0 \approx 4\pi\cdot10^{-7}\left[\frac{H}{m}\right],\quad \text{Henry} = \left[\frac{T\cdot m^2}{A}\right]$$

↑ the vacuum magnetic permiability

- The Lorentz force: $\vec{F} = q(\vec{E} + \vec{v}\times\vec{B})$

$$F_e \sim \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{r^2};\quad F_m \sim \frac{\mu_0}{4\pi}\cdot\frac{q^2 v^2}{r^2}\quad (j = q\cdot n\cdot v \sim q\cdot v)$$

$$\Rightarrow \frac{F_M}{F_e} \sim \mu_0\varepsilon_0\cdot v^2 \sim \left(\frac{v}{c}\right)^2 \leftarrow$$ magnetic forces are much weaker compared to the electric forces.

Maxwell's equations in vacuum:

$$\begin{cases} \nabla\cdot\vec{B} = 0 & \nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0} \\ \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} & \nabla\times\vec{B} = \mu_0\vec{j} + \frac{1}{c^2}\frac{\partial\vec{E}}{\partial t} \end{cases}$$

In matter, there might be induced charges (polarization) and induced currents (magnetization), that are difficult (sometimes, impossible) to measure. Therefore, we introduce

Electric displacement field: $\vec{D} = \varepsilon_0\vec{E} + \vec{P} = \varepsilon_0(1+\chi)\vec{E} = \varepsilon\vec{E}$

$\vec{P}$ - polarization density

$\vec{P} = \varepsilon_0\chi\vec{E}$, χ - electric susceptibility (linear)

Magnetic field intensity: $\vec{H} = \frac{1}{\mu_0}\vec{B} - \vec{M}\,\left[\frac{A}{m}\right]$

$$\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(1+\chi)\vec{H} = \mu\vec{H}$$

$\vec{M}$ - magnetization

$\vec{M} = \chi\vec{H}$, χ - magnetic susceptibility (linear)

Maxwell's equations in matter:

$$\begin{cases} \nabla\cdot\vec{B} = 0 & \nabla\cdot\vec{D} = \rho_f \leftarrow \text{free charges} \\ \nabla\times\vec{E} = -\frac{\partial\vec{B}}{\partial t} & \nabla\times\vec{H} = \vec{j}_f + \frac{\partial\vec{D}}{\partial t} \end{cases}$$

↑ free currents

1.1. Localized magnetism associated with ion cores

The Bohr-Van Leeuwen theorem: thermal average of the magnetization is zero:

$$\vec{F}_m = q\vec{V}\times\vec{B} \Rightarrow \vec{F}_m \perp \vec{V} \Rightarrow \underset{\text{energy}}{E(\vec{B}\neq 0) = E(\vec{B}=0)} \Rightarrow \underset{\text{partition function}}{Z(\vec{B}\neq 0)} = \sum_j e^{-\frac{E_j(\vec{B}\neq 0)}{k_B T}} = Z(\vec{B}=0)$$

$$\Rightarrow \langle E\rangle = -\frac{\partial \ln Z}{\partial \beta} = const, \quad C_V = \frac{\partial \langle E\rangle}{\partial T} = const, \quad S = \frac{\partial}{\partial T}(k_B T \ln Z) = const, \quad F = -k_B T \ln Z = const.$$

The origins of the magnetic properties of an atom:

$B=0$: $\hat{H}_0 = \sum \frac{\hat{p}^2}{2m} + U(2)$ ← energy of electrons in an atom

$\vec{B}\neq 0$: $\vec{B} = \nabla\times\vec{A}$, $\vec{A} = \frac{1}{2}[\vec{B}\times\vec{r}]$ ← in this case, $\nabla\vec{A} = \frac{1}{2}(\vec{r}\cdot\nabla\times\vec{B} - \vec{B}\cdot\nabla\times\vec{r}) = 0$,

$(\vec{B}=const)$ so $\hat{p}$ and $\hat{A}$ commutate, $[\hat{p},\hat{A}]=0$

Electrons have spin $\frac{1}{2}\hbar$ and the corresponding magnetic moment:

$$\mu_B = \frac{e}{m}\cdot\frac{1}{2}\hbar = \frac{e\hbar}{2m} = 9.27\cdot 10^{-24}[A\cdot m^2],[J/T] \quad \text{- the Bohr magneton.}$$

$$\hat{H} = \sum \frac{1}{2m}(\hat{\vec{p}} - e\hat{\vec{A}})^2 + U(2) - 2\mu_B \hat{\vec{S}}\vec{B} \qquad (\hat{\vec{S}} = \pm\tfrac{1}{2} \text{ - dimentionless spin})$$

$$= \hat{H}_0 + \hat{V}, \qquad \hat{V} = \sum -\frac{2e}{2m}\hat{p}\vec{A} + \frac{e^2\vec{A}^2}{2m} - 2\mu_B\hat{\vec{S}}\vec{B}$$

$$= \sum \frac{e^2[\vec{B}\times\vec{r}]^2}{8m} - \frac{e}{2m}\underbrace{\hat{p}[\vec{B}\times\vec{r}]}_{\vec{B}[\vec{r}\times\hat{p}]} - 2\mu_B\hat{\vec{S}}\vec{B}$$

$\vec{B}[\vec{r}\times\hat{p}]$: $\hbar\cdot\hat{L}$ - orbital angular momentum

$$= \underbrace{\sum \frac{e^2[\vec{B}\times\vec{r}]^2}{8m}}_{\text{orbital diamagnetism}} - \underbrace{\mu_B(\hat{\vec{L}} + 2\hat{\vec{S}})\vec{B}}_{\text{Zeeman energy}}$$

a) for atoms with $\hat{L}=0$, $\hat{S}=0$ $(\hat{J} = \hat{L} + \hat{S} = 0)$

$$\Delta E = \sum \frac{e^2}{8m}\cdot\langle[\vec{B}\times\vec{r}]\rangle^2 = \frac{e^2 B^2}{8m}\sum\langle r_x^2 + r_y^2\rangle = \frac{e^2B^2}{8m}\cdot\frac{2}{3}\sum\langle r_i^2\rangle \approx \frac{e^2B^2\cdot Z\langle r^2\rangle}{2\cdot 6m}$$

(size of an atom: $\langle r^2\rangle$)

$$\Delta E = -\vec{m}\vec{B} \Rightarrow \vec{m} = -\frac{\partial E}{\partial \vec{B}} \leftarrow \text{induced magnetic moment of an atom}$$

$$\vec{m} = \chi\vec{H} = \chi\frac{\vec{B}}{\mu_0} \Rightarrow \chi = \frac{\mu_0}{B}\vec{m} = -\frac{\mu_0}{B}\cdot\frac{\partial E}{\partial B} = -\frac{\mu_0 e^2}{6m} Z\langle r^2\rangle$$

b) if $\hat{\vec{J}} = \hat{\vec{L}} + \hat{\vec{S}} \neq 0$

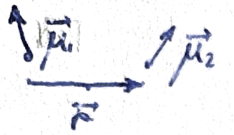
the atom has a magnetic moment:

$$\mu = \underbrace{g}_{\text{the Landé g-factor}} \cdot \mu_B \cdot J \qquad g = \begin{cases} 1, & \text{if } \hat{L} \neq 0 \text{ and } \hat{\vec{S}} = 0 \\ 2, & \text{if } \hat{L} = 0 \text{ and } \hat{\vec{S}} \neq 0 \\ 1 + \dfrac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}, & \text{if } \hat{L} \neq 0 \text{ and } \hat{S} \neq 0 \end{cases}$$

The main contribution to magnetic susceptibility χ comes from the Zeeman energy. In this case, $\chi > 0$ (Langevin paramagnetism), and temperature-dependent, $\chi \propto \frac{1}{T}$ (Curie's law)

1.2 Interaction between localized magnetic moments

Two magnetic dipoles in solid can interact via magnetic field:



$$\vec{B}_2 = \frac{\mu_0}{4\pi}\cdot\left(\frac{3(\vec{\mu}_1\vec{r})\vec{r}}{r^5} - \frac{\vec{\mu}_1}{r^3}\right)$$

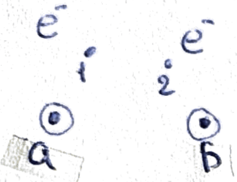
$$U = -\vec{\mu}_2\vec{B}_2 = \frac{\mu_0}{4\pi}\cdot\left(\frac{\vec{\mu}_1\vec{\mu}_2}{r^3} - \frac{3(\vec{\mu}_1\vec{r})(\vec{\mu}_2\vec{r})}{r^5}\right) \sim \frac{\mu_0}{4\pi}\cdot\frac{\mu_B^2}{a_B^3} \sim 7\cdot10^{-23}\,J \sim 5\,K$$

The moments will allign at the temperatures $\lesssim 5K$.

Dipole-dipole interaction can not explain spontaneous magnetisation up to $\sim 10^3 K$.

1.2.1 Exchange interaction

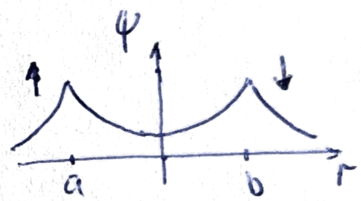
Let us consider two electrons around two neighboring atoms



electrons interact: $V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{|\vec{r}_1-\vec{r}_2|}$

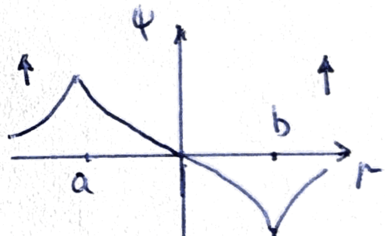
If spins are anti-parallel $(S=0)$

$$\psi_0(\vec{r}_1,\vec{r}_2) = \frac{1}{\sqrt{2}}\left[\psi_a(\vec{r}_1)\cdot\psi_b(\vec{r}_2) + \psi_a(\vec{r}_2)\cdot\psi_b(\vec{r}_1)\right]$$



If spins are parallel $(S=1)$

$$\psi_1(\vec{r}_1,\vec{r}_2) = \frac{1}{\sqrt{2}}\left[\psi_a(\vec{r}_1)\psi_b(\vec{r}_2) - \psi_a(\vec{r}_2)\psi_b(\vec{r}_1)\right]$$



Energy of electron interaction:

$$\Delta E = \iint \psi^*(\vec{r}_1,\vec{r}_2)\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{e^2}{|\vec{r}_1-\vec{r}_2|}\cdot\psi(\vec{r}_1,\vec{r}_2)\cdot d\vec{r}_1 d\vec{r}_2 = A \pm J \quad \begin{pmatrix} + \text{ for } S=0 \\ - \text{ for } S=1 \end{pmatrix}$$

$$A = \frac{e^2}{4\pi\varepsilon_0}\cdot\iint\frac{|\psi_a(\vec{r}_1)|^2\cdot|\psi_b(\vec{r}_2)|^2}{|\vec{r}_1-\vec{r}_2|}\cdot d\vec{r}_1 d\vec{r}_2 \leftarrow \text{Coulomb energy.} \quad A \sim \frac{e^2}{4\pi\varepsilon_0}\cdot\frac{1}{3a_B} \sim 1.5\cdot10^{-18}\,J \sim 10^5\,K$$

$$J = \frac{e^2}{4\pi\varepsilon_0}\cdot 2\,\mathrm{Re}\iint\frac{\psi_a^*(\vec{r}_1)\psi_b^*(\vec{r}_2)\cdot\psi_a(\vec{r}_2)\cdot\psi_b(\vec{r}_1)}{|\vec{r}_1-\vec{r}_2|}\cdot d\vec{r}_1 d\vec{r}_2 \leftarrow \text{exchange energy.}$$

if $J>0 \Rightarrow$ min. energy for $S=1 \Rightarrow$ ferromagnetic order ↑ ↑ ↑ ↑

if $J<0 \Rightarrow$ min. energy for $S=0 \Rightarrow$ antiferromagnetic order ↑ ↓ ↑ ↓

$$J \sim \frac{e^2}{4\pi\varepsilon_0}\cdot 2\cdot e^{-2\frac{r_{ab}}{a_B}}\frac{1}{a_0} \sim \frac{e^2}{2\pi\varepsilon_0 a_B}e^{-6} \sim 6\cdot10^{-21}\,J \sim 400\,K$$

$$\psi \sim e^{-r/a_B}$$

The exchange can be indirect:

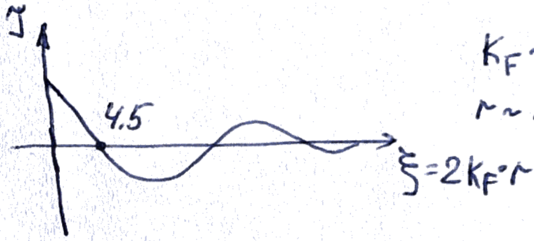
a) Superexchange in insulators (MnO):
indirect exchange between 3d-electrons of metal via 2p-electrons of oxygen

Sign of J depends on M-O-M angle: $J<0$ for $120°$-$180°$ (strong antiferromagnetic)
$J>0$ for $90°$ (weak ferromagnetic)

b) Double exchange ($La_{1-x}Sr_xMnO_3$):
indirect exchange between localized 3d electrons via delocalized 3d electrons (a shared electron between Mn^{4+} and Mn^{3+} ions)

c) RKKY interaction in rare earths (Ruderman, Kittel, Kasuya, Yosida)
indirect exchange between very localized 4f electrons via 5d/6s conduction electrons.

Sign of J depends on distance



$k_F \sim 0.1\ nm^{-1}$
$r \sim$ nanometers.

1.2.2 The Heisenberg model

For two electrons: $\hat{S}^2 = (\hat{S}_1 + \hat{S}_2)^2 = \hat{S}_1^2 + \hat{S}_2^2 + 2\hat{S}_1\hat{S}_2 \Rightarrow \vec{S}_1\vec{S}_2 = \frac{1}{2}(S^2 - S_1^2 - S_2^2) = \begin{cases} -\frac{3}{4}, & S=0 \\ \frac{1}{4}, & S=1 \end{cases}$

$S_1^2 = \frac{1}{2}(\frac{1}{2}+1)$
$S_2^2 = \frac{1}{2}(\frac{1}{2}+1)$ $\quad S=0,1$

The splitting of energy between $S=0$ and $S=1$ is $2J$. It can be written as

$$\hat{H} = -2J \cdot \hat{\vec{S}}_1\hat{\vec{S}}_2$$

Heisenberg expanded this model to multielectron atoms:

$$\hat{H} = -\sum_{i \neq j} J_{ij}\,\hat{\vec{S}}_i\hat{\vec{S}}_j - g\mu_B\vec{B}\cdot\sum_i \hat{\vec{S}}_i$$

Depending on $J_{ij} = J(\vec{r}_i - \vec{r}_j)$ there can be different types of order:

ferromagnetic, antiferromagnetic, antiferrimagnetic, spiral, etc.

↑↑↑↑ ↑↓↑↓ ↑↓↑↓↑↓ ↑↗→→↘↓↓

2. Micromagnetism and domains

Let us consider a piece of magnetic material in an external magnetic field $\vec{H}_{ext}$, in which magnetization $\vec{M}(\vec{r})$ is a smooth function of coordinates and $\vec{M}$ only changes its direction, while the magnitude $|\vec{M}| = M_s$ is constant.

2.1 Micromagnetic model (W. Brown, 1940)

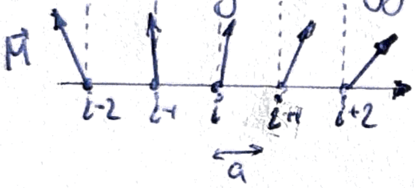
$\vec{M}(\vec{r})$ is determined by a global (or local) minimum of total energy:

$$U_{tot} = U_Z + U_{ex} + U_{an} + U_{ms} + U_{st}$$

a) Zeeman energy (magnetic moments in external field)

$$U_Z = -\mu_0 \int \vec{H}_{ext} \cdot \vec{M}(\vec{r}) \cdot d\vec{r}$$

b) Exchange energy (exchange interaction between magnetic moments)



$$E = -2J\vec{S}_1\vec{S}_2 = -2JM_s^2 \cdot \cos(\Delta\theta) \underset{\Delta\theta \ll 1}{\approx} const + JM_s^2 (\Delta\theta)^2$$

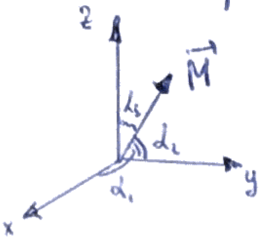
$$\approx const + JM_s^2 \cdot a^2 \left(\frac{\partial\theta}{\partial x}\right)^2$$

The main contribution to the exchange energy comes from the heterogeneity (variation) in the direction of the magnetization

$$U_{ex} = \frac{A}{M_s^2} \int (\nabla \vec{M})^2 d\vec{r}, \quad \leftarrow \text{ for a cubic lattice, } A \sim \frac{JS^2}{a} \sim 10^{-11} \left[\frac{J}{m}\right] \text{ - exchange stiffness constant}$$

c) Magnetocrystalline anisotropy

Anisotropic crystal field influences electron orbitals and the magnetisation via spin-orbit coupling. In result, magnetic energy depend on the direction of $\vec{M}$ with respect to the crystallographic axes



$\alpha_i = \dfrac{M_i}{M_S}$ ← direction cosines $(\alpha_x^2 + \alpha_y^2 + \alpha_z^2 = 1)$
$(i = x, y, z)$

$U_{an} = K_{ik}\, \alpha_i\, \alpha_k$ + higher terms

K_{ik} can be diagonalized : $K_{ik} = \begin{pmatrix} K_{xx} & 0 & 0 \\ 0 & K_{yy} & 0 \\ 0 & 0 & K_{zz} \end{pmatrix}$
(by a proper choice of axes)

in a cubic crystal: $K_{xx} = K_{yy} = K_{zz} \Rightarrow U_{an} = K(\alpha_x^2 + \alpha_y^2 + \alpha_z^2) = const$
$\Rightarrow$ higher terms must be taken into account
$(U_{an} = K_1(\alpha_x^2\alpha_y^2 + \alpha_y^2\alpha_z^2 + \alpha_z^2\alpha_x^2) + K_2\,\alpha_x^2\alpha_y^2\alpha_z^2)$

in a uniaxial crystal: $K_{xx} = K_{yy} \neq K_{zz} \Rightarrow U_{an} = K_{xx}\underbrace{(\alpha_x^2 + \alpha_y^2)}_{1-\cos^2\theta} + K_{zz}\cdot\underbrace{\alpha_z^2}_{\cos^2\theta}$

if $K_{xx} > K_{zz}$: $U_{an} = K_{xx}\cdot\sin^2\theta + K_{zz}(1-\sin^2\theta) = (K_{xx} - K_{zz})\sin^2\theta + K_{zz} = const + K_1\cdot\sin^2\theta$
the magnetization $\vec{M}$ tends to allign $\vec{M} \parallel z \Rightarrow$ easy axis

if $K_{xx} < K_{zz}$: $U_{an} = K_{xx} + (K_{zz} - K_{xx})\cdot\cos^2\theta = const + K_1\cdot\cos^2\theta$
the magnetization $\vec{M}$ tends to lie within the xy plane $\Rightarrow$ easy plane

other sources of anisotropy: shape anisotropy (negligible for spherical samples)
surface anisotropy (negligible for large samples)

d) Magnetostatic energy (magnetic interaction between the magnetic moments)

Aligned magnetic moments $\vec{M}$ create demagnetization field $\vec{H}_d$.
The energy of the moments in this self-induced magnetic field:

$$U_{ms} = -\frac{1}{2}\mu_0 \int_V \vec{H}_d(\vec{r})\cdot\vec{M}(\vec{r})\cdot d\vec{r}$$

$$\int \vec{B}\cdot\vec{H}_d\, d\vec{r} = \int \vec{H}_d\cdot[\nabla\times\vec{A}]\, d\vec{r} = \int \nabla\cdot[\vec{A}\times\vec{H}_d]\cdot d\vec{r} + \int \vec{A}\,\underbrace{[\nabla\times\vec{H}_d]}_{=\vec{j}_f = 0}\, d\vec{r} = \oint \underset{\frac{1}{r^2}}{\vec{A}}\times\underset{\frac{1}{r^3}}{\vec{H}_d}\cdot d\vec{S} = 0$$

↑ over the whole space

$$\vec{B} = \mu_0(\vec{H}_d + \vec{M}) \Rightarrow \underbrace{\int \vec{B}\,\vec{H}_d\, d\vec{r}}_{0} = \mu_0\int \vec{H}_d^2\cdot d\vec{r} + \mu_0\int \vec{H}_d\cdot\vec{M}\cdot d\vec{r} = 0$$

$$U_{ms} = \frac{1}{2}\mu_0 \int \vec{H}_d^2(\vec{r})\, d\vec{r}$$ ← integral over the whole space.

It is difficult to calculate $\vec{H}_d(\vec{r})$.

For a Homogeniously magnetized ellipsoid: $\vec{H}_d = -\hat{N}\vec{M}$, $\hat{N} = \begin{pmatrix} N_x & 0 & 0 \\ 0 & N_y & 0 \\ 0 & 0 & N_z \end{pmatrix}$

demagnetization tensor.

For a sphere: $N_x = N_y = N_z = \frac{1}{3}$

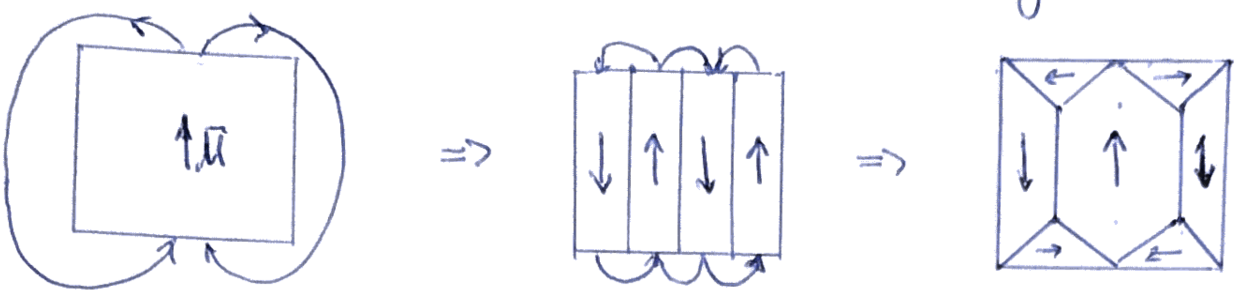
For a film: $N_x = N_y = 0$, $N_z = 1$

e) Strain and magnetostriction energy

Energy of deformation in a magnetic field.

2.2 Magnetic domains

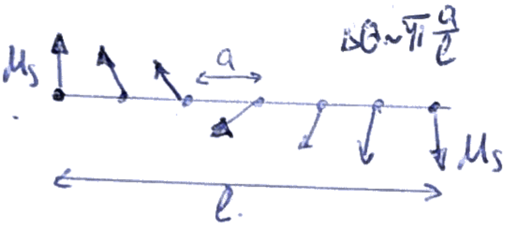
Formation of domains allows to reduce magnetostatic energy



(high anisotropy)   (low anisotropy ⇒ closure domains)

The size of the domains can be estimated from a balance between the magnetostatic energy and the energy of domain walls.

2.2.1 Bloch domain wall



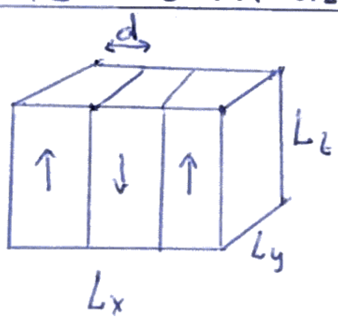
$$U_{ex} \simeq \frac{A}{M_s^2}\int_0^{\ell}\left(\frac{d\vec{M}}{dx}\right)^2 dx \sim \frac{A}{M_s^2}\int_0^{\ell}\left(\frac{M_s\cdot \pi \frac{a}{\ell}}{a}\right)^2 dx \sim \frac{A}{\ell}$$

$$U_{an} \sim K_1 \cdot \ell \qquad \text{(easy axis)}$$

$$U = U_{ex} + U_{an} \sim \frac{A}{\ell} + K_1 \cdot \ell \rightarrow \min \Rightarrow U_{ex} \sim U_{an}$$

Energy of a unit area of a domain wall: $\varkappa \sim K_1 \ell \sim \sqrt{AK_1}$ $\ell \sim \sqrt{\frac{A}{K_1}}$

2.2.2 Domain size



$$U_{walls} \sim \varkappa \cdot L_y L_z \cdot \frac{L_x}{d}$$

$$U_{ms} \sim \frac{1}{2}\mu_0 \int \vec{H}_d^2 d\vec{r} \sim \mu_0 \cdot M_s^2 \cdot L_x \cdot L_y \cdot d$$

stray field $\vec{H}_d = -N\vec{M} \sim M_s$ is only in the region $\sim d$, where domain is at the surface

$\vec{H}_d \sim 0$ in the bulk.

$$U = U_{walls} + U_{ms} \sim \varkappa_1 L_y L_z L_x \cdot \frac{1}{d} + \mu_0 M_s^2 L_x L_y \cdot d \rightarrow \min \Rightarrow U_{walls} \sim U_{ms}$$

$$\varkappa \frac{L_z}{d} \sim \mu_0 M_s^2 \cdot d \Rightarrow d \sim \sqrt{\frac{\varkappa L_z}{\mu_0 M_s^2}}$$

$$d \sim \sqrt{\frac{K_1 \ell \cdot L_z}{\mu_0 M_s^2}}$$

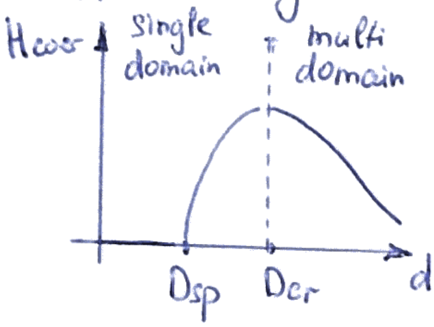
2.3 Single-domain particle

Below some critical size of a particle D_{cr}, formation of domains is unfovarable: $D_{cr} \sim \sqrt{\frac{K_1 \cdot l \cdot D_{cr}}{\mu_0 M_s^2}} \Rightarrow D_{cr} \sim \frac{K_1 l}{\mu_0 M_s^2} \sim \frac{\sqrt{A K_1}}{\mu_0 M_s^2} \sim 50-100nm$

2.3.1. Superparamagnetism

In a very small single-domain particle ($d < D_{sp} < D_{cr}$), the direction of magnetisation flips very fast on a time-scale $\tau \sim \tau_0 \cdot e^{-\Delta/k_B T}$, where $\tau_0 \sim 10^{-9}$ s, $\Delta \sim 1eV$ - anisotropy energy.

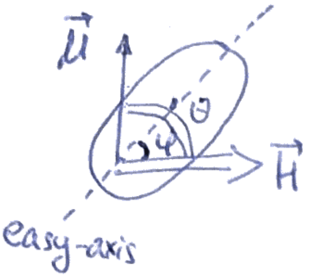
If the measurement time T_{mes} is longer than τ, the apperent magnetisation is zero: the particle appears to be paramagnetic.



Superparamagnetism limits the storage density of a HDD.

2.3.2 The Stoner-Wohlfarth model
(Stoner, Wohlfarth, Neel, 1947-1948)

Let us calculate the equilibrium magnetisation $\vec{M}(\vec{H})$ of a single-domain particle (or a single domain) in an external magnetic field $\vec{H}$



$$U = \underbrace{K \cdot V \cdot \sin^2(\varphi-\theta)}_{\text{anisotropy}} - \underbrace{\mu_0 M_s V H \cos\varphi}_{\text{Zeeman energy.}}$$

$\frac{\partial U}{\partial \varphi} = 0$ ← condition for the extremum of energy

$\frac{\partial^2 U}{\partial \varphi^2} > 0$ ← condition for the minimum of energy.

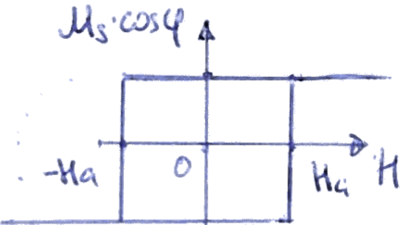
$$\frac{\partial U}{\partial \varphi} = 0 \Rightarrow K \cdot \underbrace{2\sin(\varphi-\theta)\cdot\cos(\varphi-\theta)}_{\sin(2(\varphi-\theta))} + \mu_0 M_s \cdot H \cdot \sin\varphi = 0$$

$$\frac{\partial^2 U}{\partial \varphi^2} > 0 \Rightarrow 2 \cdot K \cdot \cos(2(\varphi-\theta)) + \mu_0 M_s H \cos\varphi > 0$$

a) $\theta = 0$: $K \cdot \sin 2\varphi + \mu_0 M_s H \cdot \sin\varphi = 0 \Rightarrow \varphi = 0, \pi, \pi - \arccos \frac{\mu_0 M_s H}{2K}$

$\cos 2\varphi + \underbrace{\frac{\mu_0 M_s}{2K}}_{= \frac{1}{H_a}} \cdot H \cdot \sin\varphi > 0 \Rightarrow$ minimum at $\begin{cases} \varphi = \pi, & H < -H_a \\ \varphi = 0, \pi, & -H_a < H < H_a \\ \varphi = 0, & H > H_a \end{cases}$

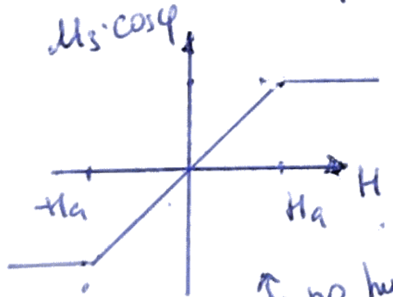
H_a - anisotropy field



↖ Hysteresis

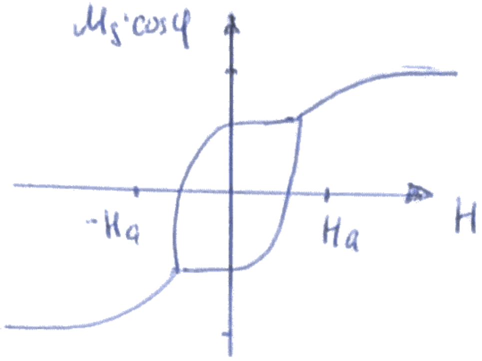
b) $\theta = \frac{\pi}{2}$: $-K \cdot \sin 2\varphi + \mu_0 M_s H \cdot \sin\varphi = 0 \Rightarrow \varphi = 0, \pi, \arccos \frac{\mu_0 M_s H}{2K}$

$-\cos 2\varphi + \frac{H}{H_a} \cdot \sin\varphi > 0 \Rightarrow$ minimum at $\begin{cases} \varphi = \pi, & H < -H_a \\ \varphi = \arccos \frac{H}{H_a}, & -H_a < H < H_a \\ \varphi = 0, & H > H_a \end{cases}$



↖ no hysteresis

c) $0 < \Theta < \frac{\pi}{2}$



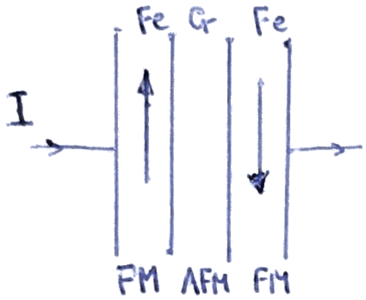
3. Magnetoresistance

$$MR = \frac{R(H) - R(0)}{R(0)} = \frac{\Delta R(H)}{R(0)}$$ ← change of the resistance in presence of a magnetic field

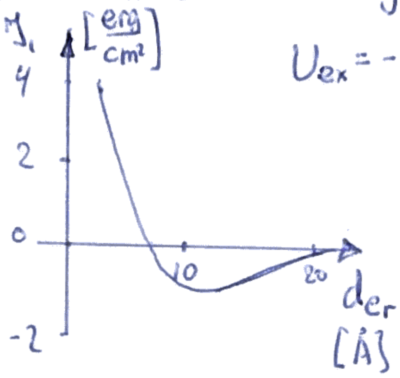
Typically $MR \lesssim 1\%$ due to change in the trajectories of electrons

3.1 Giant magnetoresistance (GMR) (Albert Fert, Peter Grünberg, 1988)

Trilayer Fe (001) / Cr (001) / Fe (001): for thin Cr layers, an antiferromagnetic coupling is observed between the Fe-layers. (RKKY-type exchange)

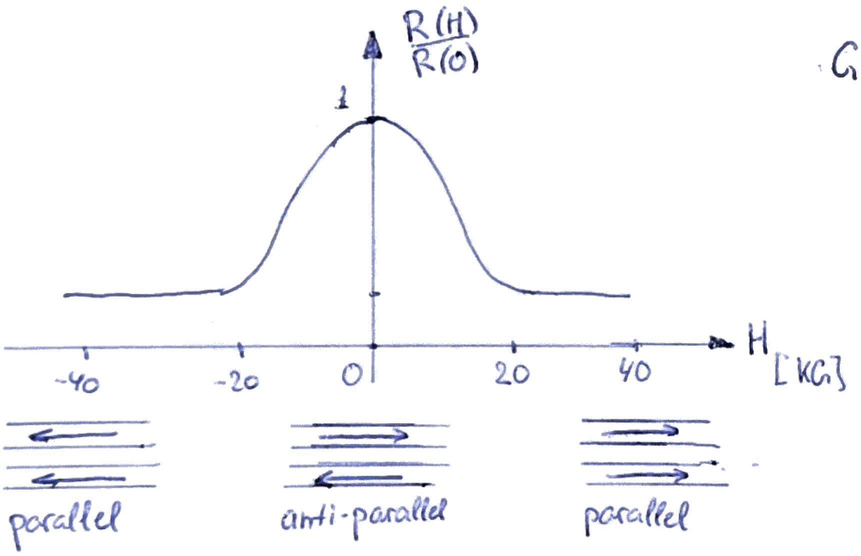


(Current perpendicular to plane: CPP-geometry)



$$U_{ex} = -J_1 \cdot \frac{\vec{M}_1 \vec{M}_2}{|\vec{M}_1||\vec{M}_2|}$$

External magnetic field can align the magnetization in Fe-layers. This has influence on resistance:



$$GMR = \frac{R_a - R_p}{R_p} = 1.5\% \quad (Fe/Cr/Fe)$$

80% (40-layers)

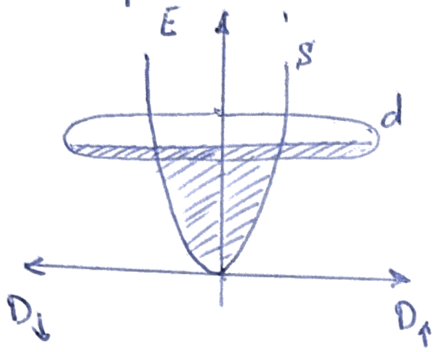
200% (40-layers at 4.2K)

applications: GMR-type sensors for HDDs

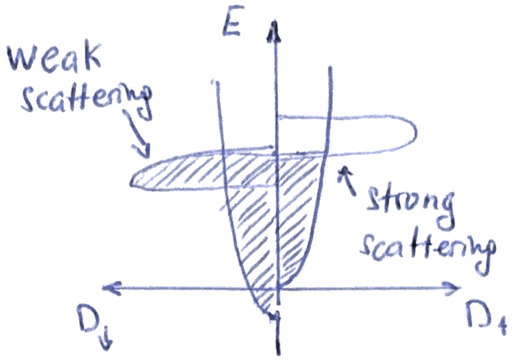
Magnetic RAM

3.2 Two-channel-model of GMR

Origin of the resistance: scattering of conducting s-electrons into unoccupied d-states.

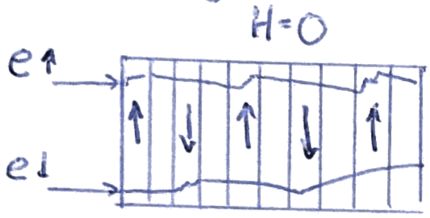


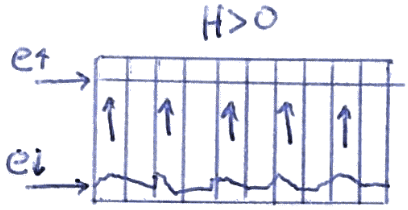
non-magnetic metal

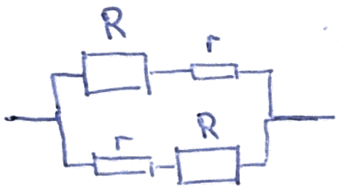


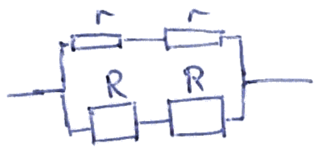
magnetic metal $(T < T_c)$
with internal $\vec{H}$.

in a magnetic metal, electrons with spins up and down scatter differently









$\Rightarrow R_{\uparrow\downarrow} > R_{\uparrow\uparrow}$