Diffusion in Drosophila Embryos

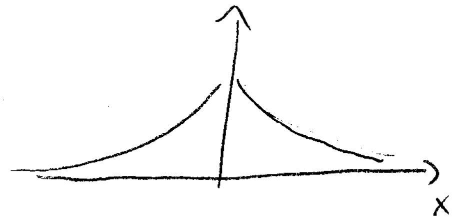
Steady state solution

$$\frac{\partial^2}{\partial x^2} P = \frac{1}{D\tau} P(x) - \frac{k}{D}\delta(x) \qquad [D] = \frac{m^2}{s} \qquad [\tau] = s$$

$P(x) \sim e^{-x/\sqrt{D\tau}}$ for $x \neq 0$ $\Rightarrow$ gradient is shallower if D is larger or the protein is more stable.

To determine the prefactor, we need to integrate over a small interval around 0

$\Rightarrow$ reflect solution around 0



$P(x)$ is non-differentiable a $x=0$

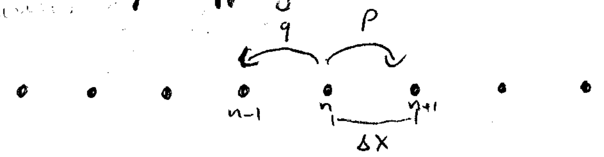
$$\int_{-\epsilon}^{\epsilon} dx \frac{\partial^2}{\partial x^2} P = \frac{2\epsilon}{D\tau} P(0) - \frac{2k}{D}$$

$$P'(x^+) - P'(x^-) = 2/\sqrt{D\tau}\, P(0) = \frac{2k}{D}$$

$$\Rightarrow P(0) = k\sqrt{\frac{D}{\tau}}$$

Two ways of getting at the diffusion equation

- limit of hopping on a lattice



$$P(n, t+\Delta t) = (1-(p+q)\Delta t)\, P(n,t) + \Delta t\, p\, P(n-1,t) + \Delta t\, q\, P(n+1,t)$$

$$\Rightarrow \quad \frac{P(n,t+\Delta t) - P(n,t)}{\Delta t} = \frac{(p-q)}{2}\Big(P(n-1,t) - P(n+1,t)\Big) + \frac{(p+q)}{2}\Big(P(n+1,t) + P(n-1,t) - 2P(n,t)\Big)$$

$$\approx -(p-q)\Delta x \frac{\partial P(x,t)}{\partial x} + \frac{(p+q)\Delta x^2}{2} \frac{\partial^2 P(x,t)}{\partial x^2}$$

$$\longrightarrow -v \frac{\partial P(x,t)}{\partial x} + D \frac{\partial^2 P(x,t)}{\partial x^2}$$

$$D = \lim_{\Delta x \to 0} \frac{p+q}{2} \Delta x^2 \qquad \left[\frac{m^2}{s}\right]$$

$$v = \lim_{\Delta x \to 0} (p-q)\Delta x \qquad \left[\frac{m}{s}\right]$$

- Chapman - Kolmogorov equation

$$P(x, t+\Delta t) = \int dy\, g(x,y,\Delta t)\, P(y,t)$$

$$= \int d\delta x\, g(\delta x, \Delta t)\, P(x-\delta x, t)$$

$$\approx \int d\delta x\, g(\delta x, \Delta t)\left(P(x,t) - \delta x \frac{\partial P}{\partial x} + \frac{1}{2}\delta x^2 \frac{\partial^2 P}{\partial x^2} + \dots\right)$$

$$= P(x,t) - v\Delta t \frac{\partial P}{\partial x} + D\Delta t \frac{\partial^2 P}{\partial x^2} + \dots$$

where $\Delta t\, v = \int dx\, x\, g(x,\Delta t)$ & $D\Delta t = \int dx\, \frac{x^2}{2}\, g(x,\Delta t)$